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Simplified Theory of the Multimode Fiber Coupler

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We simplify the coupling theory between two contiguous, parallel, multimode step-index fibers, describe the coupling concept, and derive an upper estimate for the overall coupling efficiency between the two fibers. The maximum coupling obtainable, according to this estimate, is less than 72 percent (−1.5 dB). The coupling efficiency derived for short coupling lengths shows good agreement with experimental results.

I. INTRODUCTION

A multimode fiber tap-coupler is a useful component for certain optical communication systems such as the optical data bus. Unfortunately, it is not an easy matter to evaluate the simultaneous coupling process between the hundreds or thousands of modes. An analysis of the problem has been given by Snyder and others.[1–3] These authors analyzed only the coupling between certain mode pairs. Snyder[4] recently reported the total power transition between multimode fibers. However, his conclusion is based on HE_{1m} modes for analyzing the crosstalk.

We have derived a similar simplified expression for the total coupling between identical, contiguous, parallel, step-index, multimode fibers, which can expand to all modes under the restriction that two fiber cores are touching each other. We predict that the maximum coupling efficiency is less than 1.5 dB when all modes carry an equal amount of power. The distance between two fibers affects the coupling efficiency very seriously when fibers have large numerical apertures.

Our simple formula agrees very well with experimental results, in spite of a large number of approximations made.

II. COUPLING COEFFICIENT

2.1 Simplified coupling coefficient

Figure 1 shows the geometry of the fiber coupler. The cores are parallel

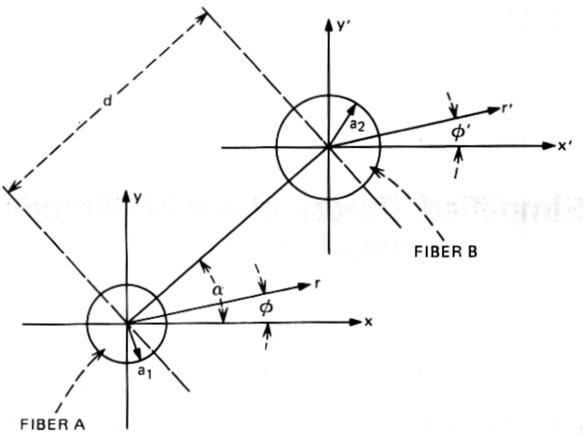

Fig. 1—Geometry used to derive the coupling coefficients.

to each other and surrounded by a medium that has the same refractive index as the cladding.

A general description of strongly coupled multimode fibers is given in Appendixes A and B. The coupling coefficient in eq. (24) has a complicated form. It involves the modified Bessel functions and many different parameters that depend upon the eigenvalue equations. We will simplify the coefficient under the following assumptions: (*i*) the two fibers are identical, (*ii*) only coupling between modes having the same propagation constant is considered (see discussion in Section 2.2), and (*iii*) the distance, d, between the two fiber axes is nearly equal to $2a$, where a is the core radius of the fiber.

We can rewrite the coupling coefficient by using assumptions (*i*) and (*ii*). The result is

$$|C_{A_iB_i}| = |C_{B_iA_i}| = \gamma \frac{\sqrt{2\Delta}}{a} \frac{u^2}{v^3} \frac{|K_{2l}(wd/a)\cos(2l\alpha) \pm K_o(wd/a)|}{K_{l-1}(w)K_{l+1}(w)}, \quad (1)$$

where

$\gamma = ½$ when $l = 0$

$\gamma = 1$ when $l > 0$

α = (defined in Fig. 1)

a = radius of core

n_0 = refractive index of core

Δ = relative refractive index difference

d = distance between the two fibers

l = azimuthal order number

$$v^2 = w^2 + u^2$$

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$$v^2 = a^2k^2n_0^2(2\Delta).$$

The coupling coefficient (1) is still complicated because it includes the modified Bessel functions. Further simplification is necessary to present a simple physical picture for the coupling process.

We use assumption (*iii*) to simplify (1) using the asymptotic expansions of the modified Bessel function. As discussed in Appendix C, the modified Bessel function term in (1) can be expressed by a very simple expression for a fairly large range of azimuthal order numbers. We use the average value of $|C_i|$ as shown in Appendix C. The result is as follows:

$$|\overline{C_{A_iB_i}}| = |\overline{C_{B_iA_i}}| = \left(\gamma\,\frac{\sqrt{2\Delta}}{a}\,\frac{u^2}{v^3}\right)\left(\frac{\sqrt{2w}}{\sqrt{\pi d/a}}\,e^{-w(d/a-2)}\right), \tag{2}$$

where

$d/a \cong 2$

$\gamma = 1$ for all modes except LP_{0m}; $\gamma = ½$ for LP_{0m}.

We let γ equal 1 for all modes. This assumption does not seriously affect the results of this analysis.

The coupling coefficient is thus,

$$|\overline{C_{A_iB_i}}| = \frac{\sqrt{2\Delta}}{a}\left(\frac{u^2}{v^3}\,\frac{\sqrt{2w}}{\sqrt{\pi d/a}}\,e^{-w(d/a-2)}\right), \tag{3}$$

where

$$d/a \cong 2.$$

The coupling coefficient expressed by (3) does not explicitly depend upon the azimuthal order number l. However, it is still dependent upon the solution of the eigenvalue equation. We simplify eq. (3) further by introducing simple expressions for u, w, and v.

The parameter v is expressed by the total number of modes in the fiber when the total number of modes is large. The results are[5,6]

$$v^2 = 2N, \tag{4}$$

where

$$v \gg 1$$

N = total mode number of fiber A and B.

We order all LP modes according to their z-propagation constants from the largest to the smallest. We label them by the sequential numbers, $i = 1 \cdots N$. For example, the two orthogonal LP_{01} modes are designated as the first and the second mode. The LP_{11} modes assume the orders 3, 4, 5, and 6. For $i \gg 1$, the cutoff value of u for the ith mode is approximately[5]

$$u_{\text{cutoff}} = (2i)^{1/2} \; (i \gg 1). \tag{5}$$

We replace u by u_{cutoff} and we obtain

$$u = (2i)^{1/2} \quad (i \gg 1). \tag{6}$$

Equation (3) can be simplified by (4) and (6). The result is,

$$|\overline{C_{A_iB_i}}| = |\overline{C_i}| = \frac{2^{3/4}}{\sqrt{\pi}} \frac{\sqrt{\Delta}}{aN^{1/4}} \left(\frac{i}{N}\right) \left(1 - \frac{i}{N}\right)^{1/4} \times \frac{\exp[-(2N - 2i)^{1/2}(d/a - 2)]}{\sqrt{d/a}} \tag{7}$$

or

$$= \frac{2^{3/4}(\Delta)^{1/4}}{\sqrt{\pi \cdot k \cdot n_0} a^{3/2}} \cdot \left(\frac{i}{N}\right) \left(1 - \frac{i}{N}\right)^{1/4} \frac{\exp[-(2N - 2i)^{1/2}(d/a - 2)]}{\sqrt{d/a}},$$

where

a = radius of core

$k = 2\pi/\lambda$

n_0 = refractive index of core

Δ = relative refractive index difference

d = distance between the two fibers.

When $d/a = 2$, the coupling coefficient becomes,

$$|\overline{C_{A_iB_i}}| = |\overline{C_i}| = \frac{2^{1/4}\,\Delta^{1/4}}{\sqrt{\pi k n_0} a^{3/2}} \left(\frac{i}{N}\right) \left(1 - \frac{i}{N}\right)^{1/4}. \tag{8}$$

This simple expression for the coupling coefficient does not require the eigenvalue solutions. Figure 2 shows that the coupling coefficient reaches a maximum for the mode number $i = (4/5)N$. The reason is the following: the coupling coefficient as expressed by (24) is based on the field interaction of the evanescent field tail of a mode of fiber A and the core field of the same mode order in fiber B. Generally speaking, the higher-order modes have a stronger field in the cladding relative to their fields in the core. Therefore, the field interaction between the field tail of a mode of fiber A and the core field of the same order in fiber B in (16)

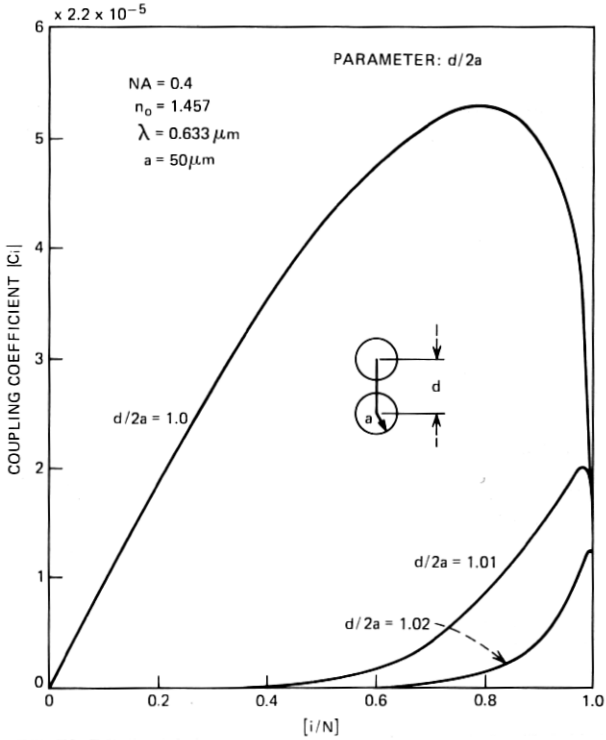

Fig. 2—Coupling coefficient vs mode order.

can be expected to increase with mode order until it reaches a maximum. Very high mode orders have very small fields in the core and therefore the interaction between very-high-order modes decreases again.

Figure 2 shows the effect of the gap between two fiber cores. Coupling occurs only between the higher-order modes as the gap increases. We will discuss the coupling efficiency using (7) and (8) in the next sections.

2.2 Coupling efficiency

The power coupled from the ith mode of fiber A to the ith mode of fiber B can be obtained from (25) when the ith mode of fiber A carries unit power and the ith mode of fiber B carries zero power at $z = 0$. The coupled power is

$$(P_{A \rightarrow B})_i = \sin^2 [|\overline{C_i}|z], \tag{9}$$

where

$$z = \text{the coupling length.}$$

The total coupled power is obtained by the summation of (9). (The coupling coefficients for some of the modes are nearly equal to zero; especially equal to zero are the coupling coefficients for one of the modes with $l = 0$. So the coupling efficiency defined here is an upper bound.) If all modes of fiber A carry equal power, then the total power from fiber A to fiber B is,

$$\frac{P_{A \to B}}{P_{in}} \leqq \frac{1}{N} \sum_{i=1}^{N} [\sin^2(|C_i|z)]. \tag{10}$$

If we treat the mode number i as a continuous variable, the coupling efficiency η becomes

$$\eta = \frac{P_{A \to B}}{P_{in}} \leqq \frac{1}{N} \int_0^N [\sin^2(|\overline{C_i}|z)di]. \tag{11}$$

The coupling between the near synchronous modes is not negligible, especially when $d/2a = 1$. However, the upper bound of the coupling efficiency is expressed by (11) when $|C_{ij}||C_{ji}| \approx |C_i|^2$ for $i - \sigma \leqq j \leqq i + \sigma$ $(\sigma/N) \ll 1$, a defined by (25). When the two fiber cores are touching each other, the coupling efficiency η is

$$\eta = \frac{P_{A \to B}}{P_{in}} \leqq \frac{1}{N} \int_0^N \sin^2 |\overline{C_i}|z\, di$$

$$= \int_0^1 \sin^2 \left(\frac{(2^{1/4}\Delta^{1/4})z}{\sqrt{\pi k n_0} a^{3/2}} t(1-t)^{1/4} \right) dt, \tag{12}$$

where

$$t = \frac{i}{N}.$$

Figures 3 and 4 show several examples.

III. DISCUSSION

As a first example, consider two fibers having the following parameters: NA = 0.2, $a = 25\ \mu$m (radius), $n_0 = 1.457$, $\gamma = 0.633\ \mu$m. This fiber carries about 1,250 modes ($N = 1{,}250$).

The coupling coefficient is

$$|\overline{C_i}| = \left(\frac{1}{\sqrt{\pi}} \frac{\sqrt{\text{NA}}}{\sqrt{ka}\, n_0} \right) \frac{(t)(1-t)^{1/4}}{a} \frac{\exp[-\sqrt{2N}(\sqrt{1-t})(d/a - 2)]}{\sqrt{d/2a}}$$

$$= 0.011 \left(\frac{1}{a} (t)(1-t)^{1/4} \frac{\exp[-2\sqrt{2N}(\sqrt{1-t})(d/2a - 1)]}{\sqrt{d/2a}} \right). \tag{13}$$

According to Fig. 2, the maximum coupling efficiency (1.5 dB) is achieved at $d/2a = 1$ and $0.011\ z/a = 3.67$. Thus, the coupling length z is about $334a$ or $z = 8.36$ mm.

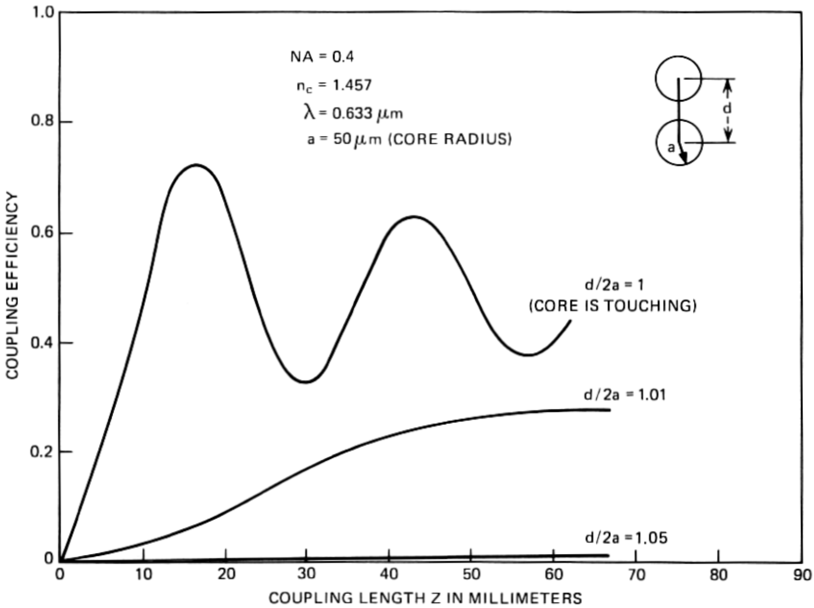

Fig. 3—Coupling efficiency vs coupling length.

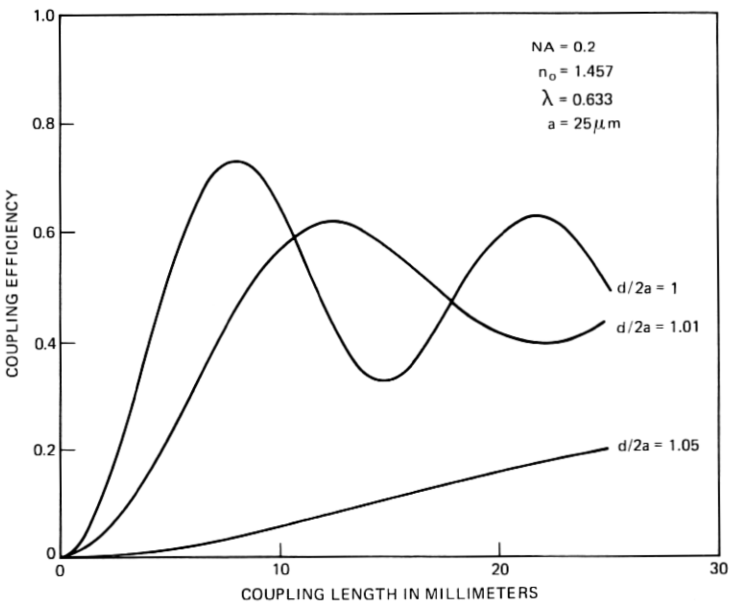

Fig. 4—Coupling efficiency vs coupling length.

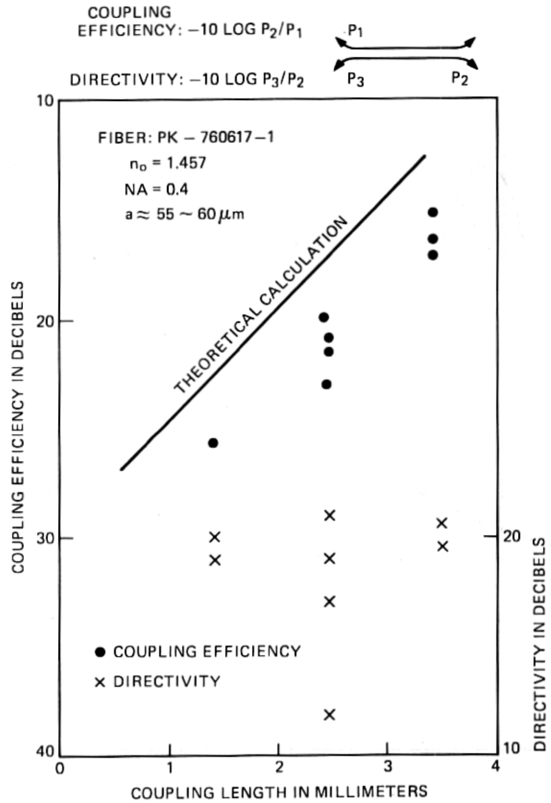

Fig. 5—Coupling efficiency vs coupling length; points indicate experimental results.

When $d/2a = 1.01$ so that the gap between the two fibers is about 0.5 μm, the coupling efficiency drops to 3.2 dB with the same coupling length (about 8.36 mm). When $d/2a = 1.05$ so that the gap between the two fibers is about 2.5 μm, the coupling efficiency becomes about 13.7 dB with the same coupling length. Thus, the distance between two fibers affects the coupling efficiency very strongly.

We now look at another example with respect to the following fiber parameters: NA = 0.4, $a = 50\ \mu$m, $n_0 = 1.457$, $\lambda = 0.633\ \mu$m. This fiber carries about 20,000 modes. The coupling coefficient is expressed by

$$|\overline{C_i}| = 0.01 \frac{1}{a}(t)(1-t)^{1/4} \times \frac{\exp\left[-2\sqrt{2N}(\sqrt{1-t})(d/2a-1)\right]}{\sqrt{d/2a}}. \tag{14}$$

The maximum coupling efficiency (about 1.5 dB) is achieved when $d/2a$

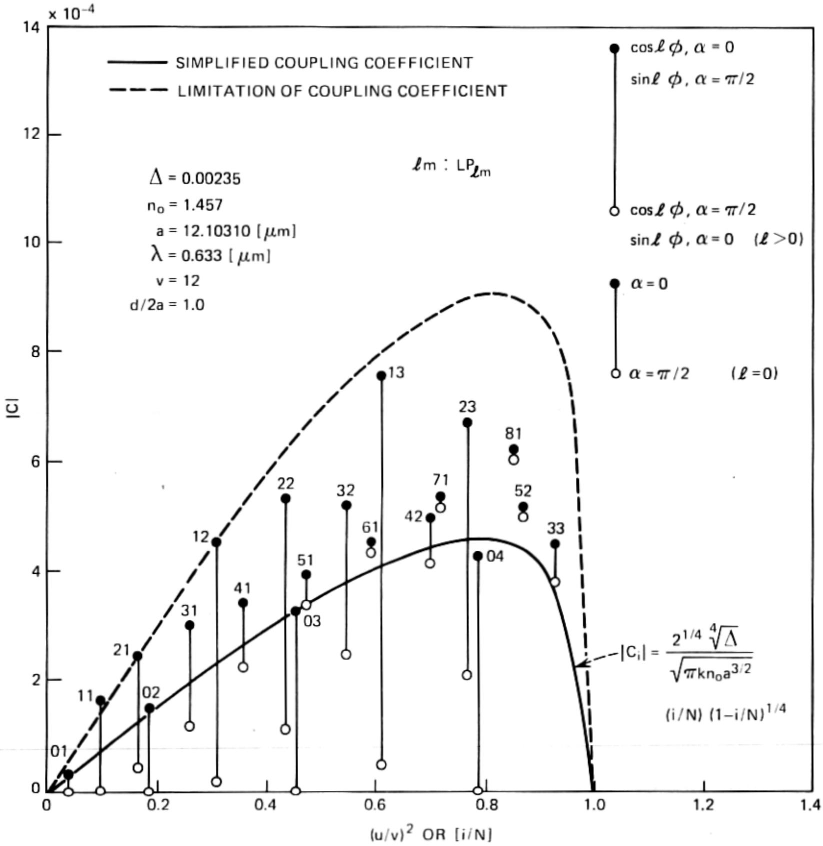

Fig. 6—Coupling coefficient vs mode order; points indicate numerical results.

= 1 and the coupling length is 16.7 mm. However, if the ratio $d/2a$ changes by only 1 percent so that the gap between the fibers becomes about 1 μm, the coupling efficiency becomes about 11.7 dB. The gap between two fibers affects the coupling efficiency more seriously when the fiber has a large numerical aperture and large radius.

Practically, it is not easy to produce a fiber coupler that has a uniform interaction gap over a long coupling length. Figure 5 shows experimental results that only cover very short coupling lengths. Plastic-clad fiber is attached to the acrylic base and its cladding is peeled off over the required coupling length. The exposed cores are pushed together and form a parallel coupling region. Silicone of the same type as the fiber cladding is injected to form a common cladding around the coupling region. Figure 4 shows that this simple theoretical approach yields good agreement. Figures 6 and 7 show results obtained from a numerical calculation of the coupling coefficient (23) and for fibers with the following parameters:

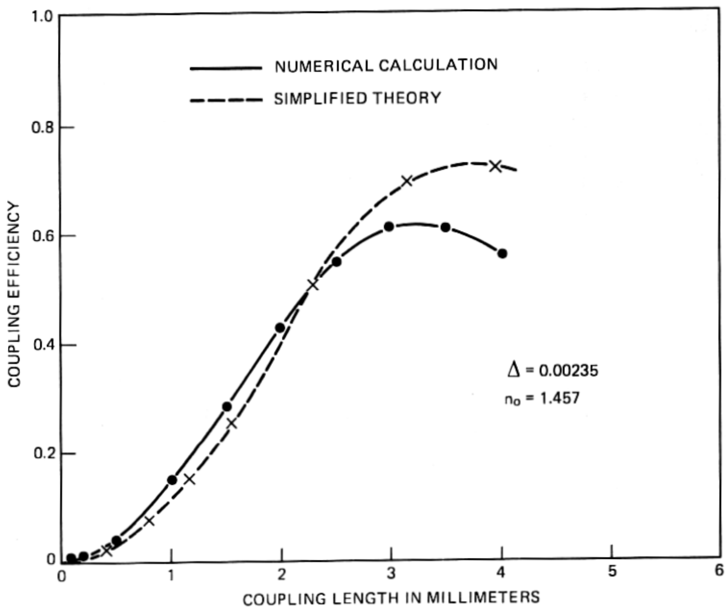

Fig. 7—Coupling efficiency vs coupling length; points indicate numerical results.

$\Delta = 0.00235$, $a = 12.1\ \mu\text{m}$, $n = 1.457$. The simplified theory shows good agreement.

IV. CONCLUSION

We have derived a very simple formula for the coupling between two multimode fibers and have discussed the coupling mechanism and the coupling efficiency. We emphasize that the formula obtained involves some rather drastic approximations. However, this coupling formula explains the coupling mechanism very clearly and agrees with experimental and numerical results.

V. ACKNOWLEDGMENT

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APPENDIX A

Coupling Coefficient

Coupling between two modes of different fibers was discussed by Snyder.[1,2] We use the formula derived by Snyder.

We assume the two fibers to be nearly identical so that the pth mode of fiber A can only couple to the qth mode of fiber B, provided the two modes have the same z-direction propagation constant.

In this case, the coupling equations have the following form

$$\frac{dA(z)}{dz} + j\beta_A A(z) = -jB(z)C_{AB}$$
$$\frac{dB(z)}{dz} + j\beta_B B(z) = -jA(z)C_{BA}, \tag{15}$$

where $A(z)$ and $B(z)$ are the amplitudes of the modes of fiber A and fiber B respectively. The coupling coefficient is defined by

$$C_{AB} = \frac{\omega}{2}\int_{\substack{\text{core area}\\ \text{of fiber } B}} \epsilon_0(n_0^2 - n_c^2)E_A E_B dS, \tag{16}$$

where

n_0 = core index of fiber B

n_c = cladding index of fibers A and B

E_A = the normalized electric field of fiber A

E_B = the normalized electric field of fiber B.

Weakly guiding fibers have simple field expressions, which are called the linearly polarized modes (LP). Each LP_{lm} mode represents a set of four modes when $l > 0$. (When $l = 0$, LP_{0m} represents two modes.) The four modes differ in polarization and azimuthal field distribution (the $\sin l\phi$ or $\cos l\phi$ term in the field-expansion equation). The field components can be described by[3]

$$E_y = H_X \begin{vmatrix} z_0/n_0 \\ z_0/n_c \end{vmatrix} = A_l \begin{vmatrix} J_l(ur/a)J_l(u) \\ K_l(wr/a)K_l(w) \end{vmatrix} [\cos l\phi \text{ or } \sin l\phi]_{r \geqq a}^{0 < r \leqq a}, \tag{17}$$

where

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}}$$

$$A_l = \left(\frac{\mu}{\epsilon}\right)^{1/4}\left(\sqrt{\frac{2}{\pi n_0}}\right)\frac{uK_l(w)}{a[v\sqrt{K_{l-1}(w)K_{l+1}(w)}]}\quad l \neq 0$$

$$A_0 = \left(\frac{\mu}{\epsilon}\right)^{1/4}\left(\frac{1}{\sqrt{\pi n_0}}\right)\frac{uK_0(w)}{avK_1(w)}\qquad l = 0.$$

These fields satisfy the following eigenvalue equation

$$u[J_{l-1}(u)/J_l(u)] = -w[K_{l-1}(w)k_l(w)]$$

or

$$u[J_{l+1}(u)/J_l(u)] = w[K_{l+1}(W)/K_l(W)]. \tag{18}$$

We calculate the coupling coefficient between the pth mode of fiber A and the qth mode of fiber B, by using (16), (17) and (18). The pth mode field of fiber A is defined by l_1, w_1, a_1, r, ϕ, and the qth mode of fiber B is defined by l_2, w_2, a_2, r', ϕ'. The coupling coefficient is defined by (17).

$$C_{A_pB_q} = \frac{\omega}{2} \frac{\epsilon_0(n_0^2 - n_c^2)}{K_{l_1}(w_1)K_{l_2}(w_2)} (A_{l1})\, A_{l2} \times \int_0^{2\pi} d\phi' \begin{Bmatrix} \cos l_1\phi\ (\cos l_2\phi') \\ \sin l_1\phi\ (\sin l_2\phi') \end{Bmatrix} \times \int_0^{a_2} r'dr'K_{l_1}\left(w_1 \frac{r}{a_1}\right) J_{l_2}\left(u_2 \frac{r'}{a_2}\right). \tag{19}$$

We introduce the following theorem to change the coordinate $(r\ \phi)$ to the coordinate (r', ϕ') shown in Fig. 1:

$$K_{l_1}\left(w_1 \frac{r}{a_1}\right) \begin{Bmatrix} \cos l_1\phi \\ \sin l_1\phi \end{Bmatrix} = \sum_{k=-\infty}^{\infty} (-1)^k K_k \left(w_1 \frac{d}{a_1}\right) I_{l_1+k}\left(w_1 \frac{r'}{a_1}\right) \times \begin{Bmatrix} \cos\,[(l_1 + k)\phi' - k\alpha] \\ \sin\,[(l_1 + k)\phi' - k\alpha] \end{Bmatrix}. \tag{20}$$

The coupling coefficient is,

$$C_{A_pB_q} = \frac{\omega}{2} \frac{\epsilon_0(n_0^2 - n_c^2)}{K_{l_1}(w_1)K_{l_2}(w_2)} A_{l_1}(A_{l_2})\pi[Q(l_1,l_2)] \times \int_0^{a_2} r'dr'I_{l_2}\left(w_1 \frac{r'}{a_1}\right) J_{l_2}\left(u_2 \frac{r'}{a_2}\right)$$

$$= \frac{\dfrac{\omega}{2} \dfrac{\epsilon_0(n_0^2 - n_c^2)\pi}{K_{l_1}(w_1)K_{l_2}(w_2)} A_{l_1}A_{l_2}[Q(l_1,\, l_2)]}{\dfrac{\omega}{2} \dfrac{\epsilon_0(n_0^2 - n_c^2)\pi}{K_{l_1}(w_1)K_{l_2}(w_2)} A_{l_1}A_{l_2}[(Q(l_1,\, l_2)][R(l_1,\, l_2)]}, \tag{21}$$

where

$$Q(l_1,\, l_2) = (-1)^{l_1 - l_2} K_{l_1 - l_2}\left(w_1 \frac{d}{a_1}\right) \cos\,(l_1 - l_2)\alpha \pm (-1)^{l_1 + l_2} K_{l_1 + l_2}\left(w_1 \frac{d}{a_1}\right) \cos\,(l_1 + l_2)\alpha$$

$$R(l_1, l_2) = \frac{a_2}{\left(\frac{u_2}{a_2}\right)^2 + \left(\frac{w_1}{a_1}\right)^2} \left[\frac{u_2}{a_2} J_{l_2+1}(u_2) I_{l_2}\left(w_1 \frac{a_2}{a_1}\right) + \frac{w_1}{a_1} J_{l_2}(u_2) I_{l_2+1}\left(w_1 \frac{a_2}{a_1}\right)\right].$$

If we substitute the eigenvalue equation into $R(l_1, l_2)$, we obtain

$$R(l_1, l_2) = \frac{a_2}{\left(\frac{u_2}{a_2}\right)^2 + \left(\frac{w_1}{a_1}\right)^2} \frac{J_{l_2}(u_2)}{K_{l_2(w_2)}} \times \left|\frac{w_2}{a_2} K_{l_2+1}(w_2) I_{l_2}\left(w_1 \frac{a_2}{a_1}\right) + \frac{w_1}{a_1} K_{l_2}(w_2) I_{l_2+1}\left(w_1 \frac{a_2}{a_1}\right)\right|. \tag{22}$$

The final result for the coupling coefficient is

$$C_{A_pB_q} = \frac{kn_0a_2}{a_1} \frac{2\Delta_2 u_1 u_2}{v_1 v_2} \left\{w_2 K_{l_2+1}(w_2) I_{l_2+1}\left(w_1 \frac{a_2}{a_1}\right) + w_1 \frac{a_2}{a_1} K_{l_2}(w_2) I_{l_2+1}\left(w_1 \frac{a_2}{a_1}\right)\right\}$$

$$\times \frac{\left\{(-1)^{l_1-l_2} K_{l_1-l_2}\left(w_1 \frac{d}{a}\right) \cos(l_1 - l_2)\alpha \pm (-1)^{l_1+l_2} K_{l_1+l_2}\left(w_1 \frac{d}{a}\right) \cos(l_1 + l_2)\alpha\right\}}{\left\{(u_2)^2 + \left(w_1 \frac{a_2}{a_1}\right)^2\right\} \left(K_{l_1-1}(w_1) K_{l_1+1}(w_1) K_{l_2+1}(w_2) K_{l_2-1}(w_2)\right)^{1/2}}. \tag{23}$$

APPENDIX B

General Coupling Equation

The coupling between two fibers, as shown in Fig. 1, is expressed by the following coupling equation with the coupling coefficient defined by (23).

$$\frac{dA_p(z)}{dz} + j\beta_{A_p} A_p(z) = -j \sum_q B_q(z) C_{A_pB_q}$$

$$\frac{dB_q(z)}{dz} + j\beta_{B_q} B_j(z) = -j \sum_p A_p(z) C_{B_qA_p}$$

$$|C_{A_pB_q}| = \frac{(\gamma_1\gamma_2)}{2}\frac{\sqrt{2\Delta_1}}{a_1v_1}u_1u_2$$

$$\times \left|\frac{K_{l_1-l_2}(w_1d/a_1)\cos(l_1-l_2)\alpha \pm K_{l_1+l_2}(w_1d/a_1)\cos(l_1+l_2)\alpha}{\left[u_2^2+w_1^2\left(\frac{a_2}{a_1}\right)^2\right]}\right|$$

$$\times \frac{(w_2)K_{l_2+1}(w_2)I_{l_2}(w_1a_2/a_1)+w_1(a_2/a_1)K_{l_2}(w_1)I_{l_2+1}(w_1a_2/a_1)}{[K_{l_1-1}(w_1)K_{l_1+1}(w_1)K_{l_2-1}(w_2)K_{l_2+1}(w_2)]^{1/2}}, \quad (24)$$

where

$A_p(z)$ = the amplitude of the pth mode of fiber A
$B_q(z)$ = the amplitude of the qth mode of fiber B
$C_{A_pB_q}$ = coupling coefficient
β_{A_p} = the z-direction propagation constant of the pth mode in fiber A
β_{B_q} = the z-direction propagation constant of the qth mode in fiber B
l_1 = the azimuthal order number of the pth mode in fiber A
l_2 = the azimuthal order number of the qth mode in fiber B
$K_l(z)$ = the modified Bessel function
a_1 = the core radius of fiber A
a_2 = the core radius of fiber B
Δ_1 = the normalized index difference between the core and the cladding of fiber A
Δ_2 = the normalized index difference between the core and the cladding of fiber B
γ_1, γ_2 = when $l_1 = 0, l_2 = 0, \gamma_1 = \gamma_2 = 1$; when $l_1 = 0, l_2 = 0, \gamma_1 = \gamma_2 = 2$.

The power transfer from the pth mode of fiber A to qth mode of fiber B is given by the following equation where the pth mode of fiber A carries unit power and the qth mode of fiber B carries no power.

$$|A_p(z)|^2 = 1 - \kappa_{AB}\sin^2\beta_{AB}z$$
$$|B_q(z)|^2 = \kappa_{BA}\sin^2\beta_{AB}z, \quad (25)$$

where

$$\kappa_{AB} = \left[1+\frac{(\beta_{A_p}-\beta_{B_q})^2}{4|(C_{A_pB_q})(C_{B_qA_p})|}\right]^{-1}$$

$$\kappa_{BA} = \left|\frac{C_{B_qA_p}}{C_{A_pB_q}}\right|\kappa_{AB}$$

$$\beta_{AB} = \left[\frac{|(C_{A_pB_q})(C_{B_q\cdot A_p})|}{\kappa_{AB}}\right]^{1/2}$$

z = the coupling length of two fibers.

Strong coupling between two modes is expected only when the propagation constants are matched. Therefore the coupling between two multimode fibers, each of which carries hundreds of modes, can be analyzed by the coupling of modes whose propagation constants are matched.

APPENDIX C

Simplification of the Modified Bessel Function

The asymptotic expansion for the modified Bessel function is given as follows

$$K_l(z) = \sqrt{\frac{\pi}{2z}}\, e^{-z} \sum_{n=0}^{\infty} \frac{(l, n)}{(2z)^n}, \tag{26}$$

where

$$|z| \text{ is large}$$
$$|\arg z| < 3\pi/2$$

$$(l,n) = \frac{(4l^2 - 1^2)(4l^2 - 3^2) \cdots (4l^2 - (2n-1)^2)}{n!2^{2n}} = \frac{\Gamma\left(l + n + \frac{1}{2}\right)}{n!\Gamma\left(l - n + \frac{1}{2}\right)}.$$

The parameters w and wd/a are large for most modes except those very close to the cutoff value. Therefore, most of the modified bessel functions can be expressed by the above asymptotic expansions. Especially when both order numbers of the mode are small (i.e., w is large), the modified Bessel function can be expressed by the first term of the asymptotic expansion. We obtain

$$\frac{K_{2l}(wd/a) \pm K_0(wd/a)}{K_{l+1}(w)K_{l-1}(w)}.$$
$$\cong 2\sqrt{\frac{2w}{\pi d/a}}\, e^{-w(d/a-2)} \qquad (\alpha = 0)$$
$$\cong 0 \qquad (\alpha = 90°). \tag{27}$$

We show below that (27) can also be used for large l. The coupling coefficient has the following modified Bessel function term:

$$\frac{K_{2l}(wd/a) \pm K_0(wd/a)}{K_{l+1}(w)K_{l-1}(w)}. \tag{28}$$

We combine (26) into (28); then the first term is,

$$\frac{K_{2l}(wd/a)}{K_{l+1}(w)K_{l-1}(w)} = \sqrt{\frac{2w}{\pi d/a}}\, e^{-w(d/a-2)} \times \frac{\sum_{n=0}^{\infty} \frac{(2l, n)}{(2w)^n (d/a)^n}}{\sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{(l+1, n-m)\,(l-1, m)}{(2w)^n}}$$

$$\cong \sqrt{\frac{2w}{\pi d/a}}\, e^{-w(d/a-2)} \times \frac{\sum_{n=0}^{\infty} \left(\frac{1}{(2w)^n}\right)\left(\frac{a}{d}\right)^n [4(2l)^2]^n \frac{1}{n!2^{2n}}}{\sum_{n=0}^{\infty} \frac{1}{(2w)^n} (4(l)^2)^n \sum_{m=0}^{n} \frac{1}{(n-m)!m!} \frac{1}{2^{2n}}}$$

$$= \sqrt{\frac{2w}{\pi d/a}}\, e^{-w(d/a-2)} \frac{\sum_{n=0}^{\infty} \frac{1}{(2w)^n} \left(\frac{a}{d}\right)^n \frac{[4(l)^2]^n}{2^{2n}\, n!} 2^{2n}}{\sum_{n=0}^{\infty} \frac{1}{(2w)^n} \frac{2^n [4(l)^2]^n}{2^{2n}\, n!}}. \tag{29}$$

If $d/a \cong 2$, the above equation is

$$\frac{K_{2l}(wd/a)}{K_{l+1}(w)K_{l-1}(w)} \cong \sqrt{\frac{2w}{\pi d/a}}\, e^{-w(d/a-2)} \frac{\sum \frac{1}{(2w)^n} \left(\frac{a}{d}\right)^n \frac{2^{2n}(4l^2)^n}{2^{2n}n!}}{\sum \frac{1}{(2w)^n} \frac{2^n (4l^2)^n}{2^{2n}n!}} \cong \sqrt{\frac{2w}{\pi d/a}}\, e^{-w(d/a-2)}. \tag{30}$$

Therefore, when l is large, the modified Bessel function term is expressed by the above equation. Then,

$$\frac{K_{2l}(wd/a) \pm K_0(wd/a)}{K_{l+1}(w)K_{l-1}(w)} \cong \xi \sqrt{\frac{2w}{\pi d/a}}\, e^{-w(d/a-2)}, \tag{31}$$

where

$\xi = 1$ when l large

$\xi = 2$ when l small, w large (even)

$\xi \simeq 0$ when l small, w large (odd).

Therefore, the coupling coefficient has the following limitation.

$$0 \leqq |\overline{C_i}| \leqq |C_{\text{limit}}| = \frac{2^{5/4}\Delta^{1/4}}{\sqrt{\pi k n_0} a^{3/2}} \left[\frac{i}{N}\right] \left[1 - \frac{i}{N}\right]^{1/4}. \tag{32}$$

However, we assume that all Bessel function terms are expressed by the case $\xi = 1$ as the average coefficient. The result is

$$|\overline{C_i}| = \frac{2^{1/4}\Delta^{1/4}}{\sqrt{k n_0 \pi} a^{3/2}} \left[\frac{i}{N}\right] \left[1 - \frac{i}{N}\right]^{1/4}, \tag{33}$$

where

$$\overline{C_i} = \text{average coupling coefficient.}$$

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